Visualization of Functionals on Freeform Surfaces

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May 1999

Abstract

A model to represent continuously differentiable functions on geometrically smooth surfaces, including trimmed surfaces, using biquadratic splines is presented and the main algorithms for the program LiLit, which has been implemented as part of this diploma thesis, are discussed.

The geometrically smooth surfaces are generated by using biquadratic g–splines. Initially, a semi–regular control net represented as a compound of polygons is oriented using the orientation of the polygons. Then the control nets for the rectangular, biquadratic Bézier patches are computed from the control points, where geometrical smoothness conditions are used near the irregularity and the standard continuity condition is used elsewhere. The semi–regular control nets can be generated by the Doo–Sabin subdivision algorithm, which can be used to refine a control net of arbitrary topology.

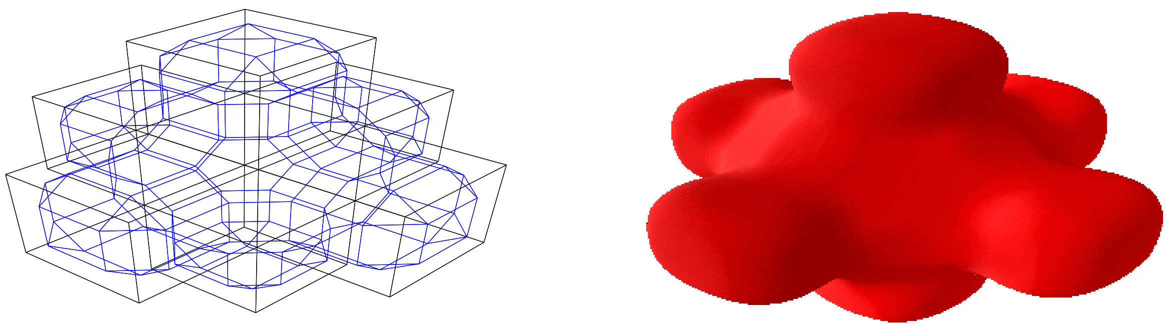

Figure 1: Doo–Sabin subdivision to create a g–spline surface

To represent a function by splines, we first consider two Bézier patches joined in a geometrically smooth manner along a common boundary curve. Two functions defined on these patches can be combined to a continuous differentiable function if they satisfy conditions formally equivalent to the geometric smoothness conditions for the patches. Thus, continuously differentiable functions can be represented as splines by adding a function control point to each surface control point. The function control points can be treated just like the surface control points by the algorithms. The functions can be visualized by spikes on the surface, by a grid over the surface, by color values on the surfaces or by color values on a surface over the surface.

Surface integrals of scalar and vector fields over the functions represented as splines can be computed as sum of the integrals of the Bézier patches. These integrals are evaluated numerically using the bivariate Romberg algorithm. In particular the area and the volume of closed surfaces can be determined.

An alternative way to represent functions on surfaces are iso–lines, i.e., lines on the surface are drawn connecting points that have the same function values.

The algorithm for this divides the parameter space adaptively using a square mesh, so that there is at most one iso–line cutting each edge of the squares. These lines can then be drawn for each square by linear interpolation. The number of lines drawn per square is limited by a maximum number of lines drawn in the smallest squares. These iso–lines are useful for drawing reflection lines on a surface. For this application, parallel light sources with a fixed distance on a plane are reflected on the surfaces using some viewing point. The points on the surface for which the reflection ray hits the same light source belong to the same iso–line.

A surface and with it the functions defined on it can be trimmed by volumes in the space of the surface as well as by curves in parameter space. The volumes are described implicitly by quadratic forms, which always cut out parts of the surface. The curves in parameter space are bound to a control point of the biquadratic surface. They can be described as a polygon or, for geometrically smooth, quadratic curves, by control points.

The spline curves are transformed into polygons, where it is assumed that the points of the polygon are traversed clockwise. If the spline curve is not closed, it is expanded suitably by the boundary of the parameter space. If the trim areas in parameter space overlap, a point is cut out if it is part of an odd number of trim areas. The trimming algorithm divides the parameter space of a Bézier patch adaptively by a square mesh. Depending on how many corners of a square are cut out, the square is either handled like in the non–trimmed case, or only a triangle is handled, or the square is cut out completely.

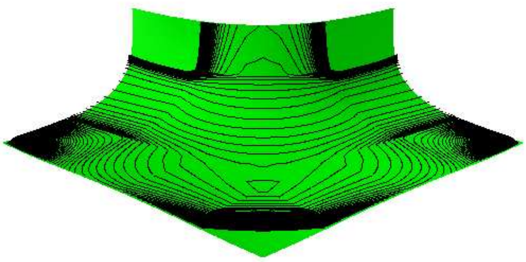

Figure 2: Reflection lines close to an irregularity of order 5

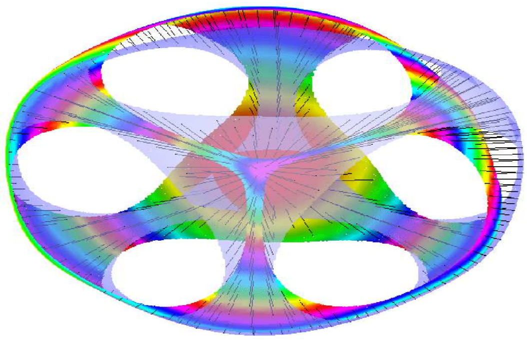

Figure 3: Function on a trimmed surface