Estimate of Frequencies of Geometric Regularities for Use in Reverse Engineering of Simple Mechanical Components

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Abstract. The authors are investigating reverse engineering for reconstructing the shape of simple mechanical parts. Currently, preliminary B-rep models can be created by fitting surfaces to point clouds obtained by scanning an actual part using a 3D laser scanner. The resulting model, although valid, is often not suitable for purposes such as redesign. This is because expected regularities and constraints are not present in the model. This report describes a number of aspects of the geometry of mechanical parts which should be exploited to adjust a B-rep model to improve its usefulness. Aspects considered are geometric constraints between surface parameters, regularly repeated substructures, symmetry, and the presence of features such as slots and holes. The results of a survey of a range of mechanical parts are presented and discussed, showing which of these aspects occur with a frequency that justifies their use in beautification algorithms intended to turn preliminary reverse engineered B-rep models into models engineers expect.

Keywords: Part Survey; Beautification; Reverse Engineering; Solid Modelling.

1 Introduction

Reverse engineering is a topic of current interest in computer aided design. Here we take it to mean recovering a CAD model of the shape of a mechanical component's boundary. A broader definition could include the determination of surface finish, chemical composition, heat treatment, or even design intent. Unlike conventional engineering, which begins with a description of what the part will do and produces a geometric model suitable for manufacturing it, reverse engineering begins with the manufactured part itself and produces a geometric model of it [19]. Typically this process involves scanning the surface of the object in order to produce a solid model. The authors' current project concerns B-rep models that have been derived from fitting surfaces to a point cloud obtained from a commercial 3D laser scanner.

Reverse engineered models typically are constructed by extracting each face separately and stitching the faces together. However, designed objects usually display constraints, features and regularities. Our long term goal is a system which will analyse a preliminary reverse-engineered model for the approximate presence of these, and will enforce some set of them on the model to produce an improved model. As groundwork for this, we need to know which constraints, features and regularities should be sought and imposed. Current reverse engineering methods can handle objects of limited complexity and geometry (see later). The main contribution of this report is a survey of a range of objects meeting those limits to determine which constraints, features and regularities commonly appear in practice.

Before conducting the survey we reviewed current literature. Mathematical work on symmetry groups in space [9] shows that that Platonic solids, prisms, pyramids, saw[1] prisms, saw pyra-

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[1] i.e. based on an n-fold circular saw shape

mids, spheres, cylinders and cones, exhibit all possible 3D symmetries. This enabled us to limit our search to particular symmetries without danger of missing any. Our work involves the detection of regular relations between elements of an object, which can be expressed by geometric constraints [3]. We are also interested in the relationships between features, but we do not intend to work on feature detection itself. Known techniques for feature detection [12] could be incorporated into our algorithms to obtain a practical system. Thus it is important that in defining our features we limit ourselves to simple features which can be detected by known algorithms of acceptable efficiency [15, 18].

The current state of the art [10] for surface fitting can reliably fit planes, spheres, cylinders, cones and tori and detect fixed-radius rolling ball blends between them [5]. Many mechanical parts can be described by these surfaces [16]. Simple parts constructed from them often exhibit symmetries and other regularities that make reverse engineering easier. This report considers only shapes made from these simple surfaces, possibly having fixed radius blends between them. Later work may include a mixture of simple and free-form surfaces.

Our reverse engineering process begins by building an initial B-rep model by scanning a physical object and then fitting a collection of surfaces to the resulting point cloud. Due to variation of the physical object from the intended design, errors in scanning, and vagaries of surface fitting, small errors creep into the model. These errors can easily shift the object from strict conformance with a regularity or constraint. In practice the inaccurate model created by surface fitting may be suitable for copying the object. However, it is unlikely to be suitable for redesign because the faces are fitted individually and so the model will only by chance reflect natural interrelations between features of the object.

Beautification is the process of adjusting the model to more closely reflect the intended structure. This will almost certainly involve regularities, features and constraints which may be aesthetically desirable, required for the part to function or to be easily manufacturable, or they may facilitate the analysis of the part.

In order to fit the collection of surfaces, typically the point cloud will be segmented and a separate surface fitted individually to each subset of points [1]. Once the basic model has been obtained, a refined model that satisfies the constraints can be produced either by re-fitting the surfaces with constraints prior to B-rep model building [2, 20] or by adjusting the faces as a post-processing step after model building.

We take the post-processing approach, which has a lower requirement for numerical computation. There is no point fitting a surface to a greater accuracy than the distance by which it might later have to be adjusted, to produce, for example, a given inter-plane distance. A tuning parameter can be provided to decide just how accurate it is necessary to make the fit.

We aim to beautify the model by finding aspects of the B-rep model that suggest the existence of regularities in the object that have been lost in the process of scanning and surface fitting. By computing certain numbers which indicate how close the object is to being regular, we are able to detect approximate regularity. We can tell if a small shift in the structure, for example a slight reorientation of a plane, would cause the constraint to be met. A new model can then be constructed that has these regularities, or satisfies related constraints. This new model is expected to be a better representation of the object.

The purpose of the survey, which is the main result of this report, is to estimate the frequency of occurrence of our particular selection of regularities which are seen as being of relevance to beautification. The intention of these estimates is to establish a qualitative classification of regularities as rare, common, or in-between. The authors are not aware of any other survey obtaining these frequencies, or of a survey that determines the individual frequencies of any other set of regularities that could be used for beautification. Nevertheless other part surveys with some relevance have been conducted. One determined the difficulty of describing parts in the CSG based language PADL [16], where the primitives had planar, spherical, conical, cylindrical and toroidal surfaces. It mentions that 93% of the parts examined were describable by cylinders, prisms and wedges. However, this was limited to the examination of the components of one brand of photocopier.

Many other areas of research have some bearing on topics in this report. For example, the University of Pennsylvania GRASP lab [25] has done work in this area; in particular the PhD thesis of Pito [14] considers acquiring and triangulating point clouds. Thompson et al. [17] worked on driving reverse engineering using feature recognition algorithms. Both Kramer [6] and Hoffmann [4] have worked on the solution of constraint systems by various methods of ge-

Surface	Generator (a)	Geometric (b)	Numerical (c)
Plane	line orthogonal to axis	normal axis	distance
Sphere	circle with axis through centre	centroid	radius
Cylinder	line parallel to axis	apex, axis	radius
Cone	line intersecting axis	centroid, axis	semi-angle
Torus	circle with axis outside circle	axis	major and minor radii

Table 1: Primitive Surfaces and their Geometric and Numerical Characteristics

ometric and combinatorial reasoning.

This report provides the ground work for beautification methods by determining which symmetries and regularities exist in practice. The survey results show roughly the frequencies in practice. Work conducted by the authors has produced efficient algorithms for finding these regularities and symmetries [7, 8, 13].

The rest of the report is divided in four sections. The first three describe the constraints, features and regularities of interest with reference to concrete examples. The last section presents and discusses the results of the survey.

2 Constraints

Classical families of geometric solids have certain parameters associated with them that can be used to specify a particular member of the family. For example, for a finite cylinder these include its length, radius, cross sectional area and volume. Some of these parameters are related by formulae, while others are independent. Specifying relationships, for example that the radius must have a set length, constrains the solid to belong to a sub-family of geometric solids. The system envisaged by the authors will determine such constraints between the parameters of one or more faces of an object. There is a vast range of formulae that could be considered. Even restricting them to rational functions can be impractical since the difficulty of dealing with these algorithmically is high [4]. Because of this, we only use a very limited class of numeric constraints, and these are listed explicitly in this section.

The surface geometries of interest are spheres, planes, cylinders, cones and tori. (We ignore blends for most purposes of analysis and beautification, as they are derived from the primary surfaces: we only have to adjust the radii of the blends, if necessary, to desired values.) It is a familiar notion that these surfaces can all be described by rotating a planar generating figure about an axis in that plane, as listed in Table 1(a). We take advantage of this in the following description of surface characteristics.

The rotation axis of each cone, torus, and cylinder is unique and is a characteristic of each surface. Toroidal surfaces also have a characteristic *circle of rotation* which is mapped out by the centre of the generating circle as it is rotated about the axis. A sphere has no special axis, but being a finite surface it has a centroid, as does a torus. We distinguish both of these as special points. For a cone, the apex which is at the intersection of the generating line and the axis serves as a special point. A plane has no distinguishable points or lines, but by introducing an origin into space we can determine a unique line through the origin and orthogonal to the plane. These special points and axes which we use to characterise the surfaces are listed in Table 1(b).

The special lines and points do not fully determine the surface. For a cylinder we need to specify its radius, which is the distance between the axis and the generator, for a cone: the smaller angle between the generator and the axis, for a torus: the radii of the generating circle and the circle of rotation, for a plane: the distance from the origin, and for a sphere: its radius. These numbers are listed in Table 1(c).

The axis of the torus, cone and cylinder rotate and translate with the surface as though physically attached. Such an axis will be called a *spatial axis.* The axis of a plane rotates, but does not translate with the surface, and is only dependent on its orientation. Such an axis will be called an *orientation axis.* For each surface with a spatial axis we also produce an orientation axis by finding the line through the origin and parallel to the axis. Each surface except the sphere has an orientation axis. Each surface except the plane has a spatial axis.

The numerical constraints considered here are mostly compounds of equality of parameters, lengths or distances being ratios of small integers, and angles being simple rational multiples of π. Congruence of faces may be expressed by equality of sufficient parameters, including lengths of edges and various angles. The con-

Regularity	Frequency
Equal radii	85.7%
Equal edge length	69.1%
Congruent faces	42.6%
2:1 radius ratio	3.2%
2:1 edge length ratio	1.8%
3:1 edge length ratio	1.4%
3:1 radius ratio	1.4%
4:1 edge length ratio	0.9%

Table 2: Shape Constraint Frequencies

Regularity	Frequency
4 or more sided pyramid vertex	1.2%
Mixed diagrams are equal	56.2%
Mixed axis angles are $m\pi/2$	47.9%
Partial centricity	83.4%
Single orthogonal system	65.9%
Faces with identical special points	65.2%
Inter axis angles $m\pi/2$	46.3%
Faces with identical special lines	42.2%
Inter axis angles $m\pi/4$	35.3%
Inter axis angles $m\pi/6$	19.3%
Main axis in an orthogonal system	18.4%
Having a main axis	13.8%
Multiple orthogonal systems	1.2%
Inter axis angles $m\pi/5$	0.9%

Table 3: Axis Constraint Frequencies

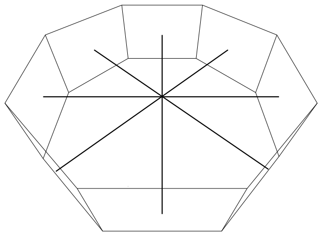

Figure 1: Orientation Regular on a Plane

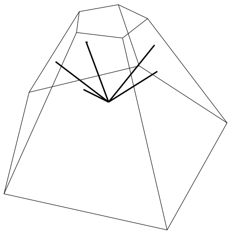

Figure 2: Orientation Regular on a Cone

dition of a collection of planes having a common point or line of intersection can be expressed in terms of certain determinants being zero. For a list of simple constraints see Table 2. More complicated constraints are discussed below, and listed in Table 3.

Although for algorithmic processing all the constraints are expressed in terms of algebraic formulae, some are best described here geometrically by the way in which axes and points are arranged in space. These axes can be constrained to lie on a common plane, cone or cylinder, with constant inter-axis distance or angle, as appropriate. The constraining surface would typically not be a surface occurring in the B-rep model itself. Such an auxiliary surface will be termed here a *ghost* surface.

A collection of circles lying on a cylinder with a constant distance between adjacent circles will be said to be *distance regular* on a ghost cylinder. A collection of several parallel coplanar lines with constant distance between adjacent lines will be said to be distance regular on a ghost plane. Coplanar lines with a common point, and constant angle between adjacent lines will be said to be *orientation regular* on a plane (see Figure 1). A collection of lines can analogously be orientation regular on a cone (see Figure 2) or distance regular on a cylinder. These ghost surface regularities and their frequencies are summarised in Table 4. Examples of these regularities occur in Figures 3, 4 and 5.

In Figure 3 the orientation axes of the four oblique surfaces containing six oblique faces at the top of the object are regular on a ghost cone. The axes of the four small holes are regular on a ghost cylinder. The axes of either pair of two diametrically opposed holes, together with the axis of the large hole, are distance regular on a ghost diagonal plane. Figure 5 shows a chuck with three slots and three symmetrically placed cylindrical holes. The axes of these holes are angle regular on a ghost plane, with a inter axis angles being of $2\pi/3$.

Ghost planes, cones and cylinders themselves have axes and centres. A constraint between an axis or special point of a ghost surface and an axis or point of a real surface in the B-rep is called a *mixed* constraint. As an example in Figure 3, the ghost cylinder containing the axes of

Regularity	Frequency
Axes regular on a cone	76.3%
Axes angle regular on a plane	63.8%
Axes regular on a cylinder	26.2%
Mixed axis angles are $m\pi/4$	23.9%
Axes distance regular on a plane	57.3%
Mixed axis angles are $m\pi/6$	5.9%
Circles regular on a cylinder	2.9%

Table 4: Regular Axis Arrangement Frequencies

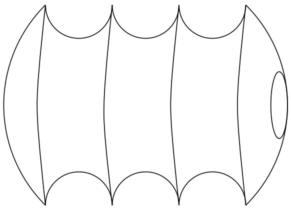

Figure 4: Example Part 2

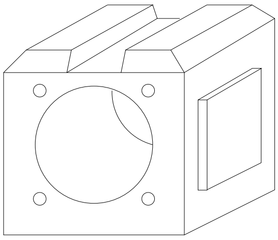

Figure 3: Example Part 1

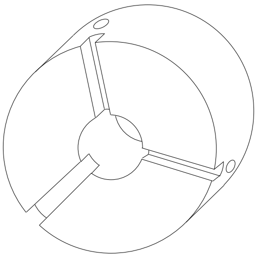

Figure 5: Example Part 3

the four small holes has an axis which is identical with that of the large hole so this object satisfies a mixed constraint.

An object is said to have an *orthogonal system* when there are many lines, axes and planes, most of which are lined up to some set of orthogonal co-ordinate axes. This is the case in Figure 3. To say that there is a main axis is to say that most of the interesting axes and planes are parallel to a given special axis. This occurs in a strict form in Figure 4. Figure 3 is a marginal case of an orthogonal system. The axes parallel to the axis of the large hole are not in the majority, even though this direction lines up with more axes than any other direction.

Figure 4 shows a pulley wheel. The three toroidal tracks are congruent, and their circles of rotation are regular on a ghost cylinder. The two end caps are part of a single spherical surface, which has its centre identical with the centre of the middle toroid. The cylindrical axle hole, drilled through the middle, has an axis which is identical to the axis of each of the toroidal surfaces.The main axis of the wheel is contained within the single orthogonal system.

Certain special points of edges and faces are examined to see if they have significant relationships to special points of other edges and faces. For each edge that has two distinct end points, the mid point between these end points is a special point called the *compound centre* of that edge. If an edge is a circular arc then the centre of the unique circle that contains that arc is a special point of the edge called the *geometric centre* of that edge. The compound centre of an edge is unique if it exists and the same for the geometric centre. When they both exist, the compound and geometric centres are not typically coincident, but this can occur. Similar but not identical definitions apply for geometric and compound centres of a face. For a face which lies on a sphere, the centre of that sphere is the geometric centre of the face. For a face which lies on a torus the centroid of that torus is the geometric centre of the face. The geometric centre, if it exists, is unique. A face may have a number of compound centres. For each simple loop formed from edges in the boundary of a face, the average position of the vertices in that loop is said to be a compound centre of the face. Thus a face has as many compound centres as it has (inner or outer) boundary loops.

A special point is said to be *partially centric* on a given straight edge or flat face, if the closest point on that edge or face to the special point is a compound centre of the edge or face (see Figure 6). For example, in Figure 3, the front face has six compound centres, one for each of the holes, and one for the outer rectangular loop of edges. The geometric centre of the larger circle is not the same as the compound centre of the rectangular loop, so this circle is not central to that loop. But the closest point on the lower straight edge to the centre of the circle is the mid point between the two end points. This is the compound centre of the lower edge. Thus

Exemplar	Regularity	Frequency
Pot-hole	Negative cylinder	72.8%
Pocket	Negative orthogonal polyhedron	28.1%
Slot	Negative prism	18.8%
Mound	Positive orthogonal polyhedron	4.1%
Ridge	Positive prism	3.2%
Stud	Positive cylinder	0.7%

Table 5: Feature Frequencies

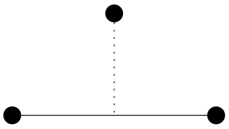

Figure 6: Partial Centricity

the large circle is partially centric to the lower edge of the outer loop of the front face.

The collection of all the spatial axes and special points associated with a face is called the *spatial diagram* of the face. A compound face has a spatial diagram which is the union of all the spatial diagrams of its components. The *orientation diagram* of a (possibly compound) face is defined analogously as a collection of orientation axes.

A constraint between a characteristic of a real face and a ghost face is called a *mixed* constraint. The terms *mixed axes* and *mixed diagrams* simply means axes or diagrams taken one from a ghost surface, and one from a real surface. For the list of mixed constraints, main axes, orthogonal systems, and compound face constraints, which were sought in the survey see Table 3.

3 Features

It is now widely agreed [12] that there is no common notion of exactly what a feature is. Intuitively, a feature is an interesting part of an object, which can be viewed in isolation, or in relation to other features. It is part of the way in which the object may be decomposed in order to be understood. However, the decomposition will depend on whether, for example, it is considered as a component in an assembly, or a product to be manufactured. This ambiguity can be cleared up by expressing the method used to decompose the object.

3.1 Feature Types

We express the mechanical component as a main body from which some material is removed, and some added. The amount removed is broken into a number of solid bodies, each being one negative feature. The amount added is broken into positive features. Each feature is determined by cutting out a part of the boundary of the main body, and gluing into its place a part of the boundary of another solid. By convention the feature is said to have the shape of the secondary solid, rather than the shape of the actual material added or removed. For example, Figure 3 shows a cube with a chamfered top into which a rectangular slot has been cut. Note that the material removed to make the slot has skewed end faces, and so is not actually a rectangular prism.

A *stud* is a positive cylindrical feature, attached by the boundary of one of its circular end faces to a circle cut from a single planar face of the object. A *pot-hole* is the negative equivalent of a stud. We do not have a through hole feature. In practice, laser scanners are not good at capturing points inside deep concavities, and in many cases would be unable to distinguish these from through holes. In the same spirit, when carrying out our survey, we treated through holes as pairs of opposing blind holes.

A *ridge* is a positive feature with the shape of a rectangular prism. Each ridge is attached primarily to a particular face of the object, by a part of a face of the prism. However it is also attached at one or two of its ends by a possibly skew plane cutting through the prism. A *slot* is the negative equivalent of a ridge, and there is one on the top of the square component illustrated in Figure 3.

A *mound* is a positive orthogonal polyhedral shape, attached to a single face of the object by exactly one of its own faces. A simple case is illustrated on the right in Figure 3. A *pocket* is the negative equivalent of a mound.

Feature types are summarised in Table 5.

3.2 Inter-Feature Constraints

A feature, negative or positive, can be seen as an object in its own right. It may have regularities, and it may be useful to continue the analysis of the feature by decomposing it further. Alternatively, a feature may have preferred directions or interesting points. In this sense it can be treated in the same way as a primitive face. For example, we can look at the relationship between the main axis of a square hole, composed of four faces, and the normal axis of some other face.

The features mentioned here are not all those that are possible, and later expansion could take in more sophisticated features. These would also have axes and points that could be analysed in the same way.

4 Symmetries and other Global Regularities

In this section we explain what symmetries and other global regularities we look for in our survey. The possible symmetries of a finite three dimensional solid can be usefully classified into four classes which will be given here the mnemonic, but not entirely accurate, names *Platonic*, *prismatic*, *pyramidal* and *anti-prismatic.* These classes overlap, but this will not concern us here. The motive is to produce exemplars for all the possible symmetries a mechanical component might have.

The Platonic symmetries are the symmetries of roughly ball-like solids. These include the spherical, cubical, icosahedral, dodecahedral, tetrahedral and octahedral symmetries. An object with a Platonic symmetry does not have to look like a Platonic solid, but the repetition of its parts has to be in the same pattern. The Platonic solids are the simplest solids with Platonic symmetry.

A prism on a given base is simply an extrusion with the base as cross section. The symmetries of the prism depend only on the order of the rotational symmetries of the base and whether or not it is mirror symmetric. A prism on a symmetric base has symmetries: rotation about the main axis, two-fold rotation about orthogonal axes, mirror reflection in planes containing the main axis and in one plane orthogonal to the main axis, and inversion through the centre of the prism. These are the full prismatic symmetries. The planes and axes involved are shown in Figure 7. If the base is not mirror symmetric, then the two-fold rotations, and the main axis mirror symmetries do not occur.

A pyramid is produced by shrinking the cross section as it is extruded. This does not affect the main axis rotation or reflection symmetry, but does remove the orthogonal reflection symmetry, the two-fold rotations about orthogonal axes, and the inversion through the centre. If the base is not mirror symmetric, then the main axis reflections do not occur either. A pyramid on an non mirror symmetric base has only the main axis rotational symmetry.

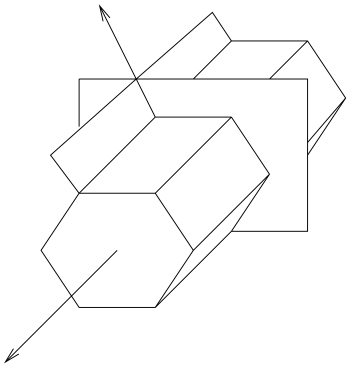

Figure 7: Prism showing Centre Lines

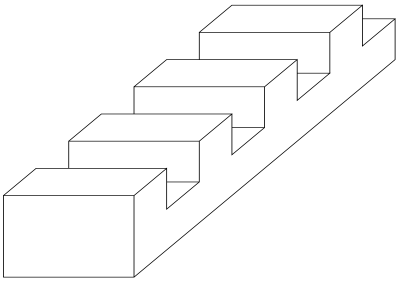

Figure 8: Rack of Teeth

Anti-prismatic symmetry is exhibited by an object made up from two copies of the same prism, aligned on the same axis, and rotated by $\pi/(2n)$ radians with respect to each other. The two-fold rotations are prevented by the twist, and there is no orthogonal mirror plane. But a combination of a horizontal reflection, and a rotation about the main axis is still a symmetry of the object.

As mentioned above, a mechanical component with one of these symmetries does not have to look like the exemplar. However, each such component can be broken up into pieces that relate in the same way. So, for example, a component with cubic symmetry can be seen as eight congruent pieces, corresponding to the octants of a cube, where each piece is made of three parts, related by a rotation. Similar examination determines the number of parts, and the relationship between the parts for each of the other symmetries.

The symmetries of a regular prism based on a polygon of n sides can be generated as combinations of the following operations. It may be rotated about its main axis by any multiple of $2\pi/n$, or reflected in any of $2n$ planes containing the main axis, or reflected in the plane orthogo-

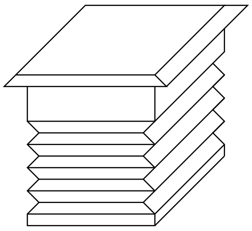

Figure 9: Typical Plastic Cap

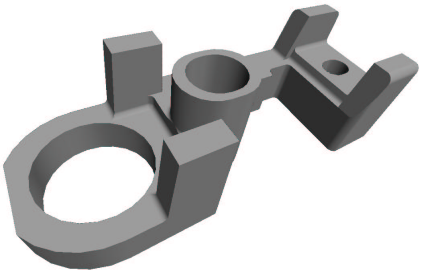

Figure 11: Gehäuse Part

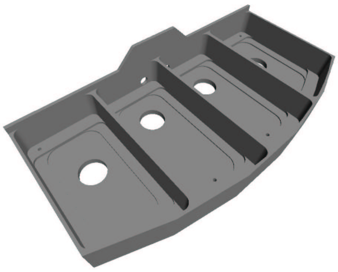

Figure 10: Part included in the Survey

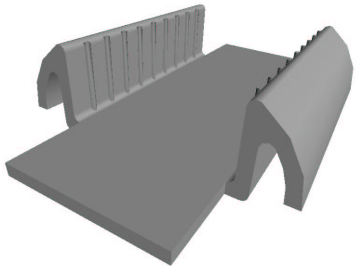

Figure 12: Heatsink Part

nal to the main axis and containing the centroid, or it may be turned over about an axis through the centroid, and orthogonal to the main axis. These planes and axes are indicated in Figure 7 for a six sided prism. A complete description of all the possible symmetries of finite solids is well known [9].

Sometimes an object will be made of a suggestive collection of congruent pieces, even though the object itself is not symmetric. For example, a rack of teeth (see Figure 8) is made up from a collection of copies of a single tooth. If the movement that takes the first tooth onto the second was repeated, then eventually all of the teeth would be generated. This type of regular arrangement in space is not accidental and is just as important as a full symmetry. Detection of this type of incomplete symmetry has many algorithmic similarities to detection of full symmetry. In this report we consider two basic types of incomplete symmetry. Firstly the regularity of a rack of teeth, with the special case of an extrusion in which the number of segments is infinite, and then the analogous incomplete cog-wheel, and the pie section special case.

The models actually input to the beautification algorithms are expected to be non-regular because the scanning and face fitting process introduces errors in the location of the faces, and because the part itself will have features, such as bolt holes, that spoil the regularity. The part in Figure 3 is only cubical if large features, such as the middle hole, are ignored. A regularity will be considered to be present if it is possible to meaningfully decompose the object into a main body, which exhibits the regularity, and a number of features, that might not. The less features that have to be excluded from the main body, the more significant the regularity.

Detection of the regularity will be based on numerical approximation rather than exact congruence. It is unlikely that, for example, the front and rear faces fitted to the scanned point cloud would actually be parallel immediately after the initial face fitting. The definitions of approximation, and the algorithms for determining approximate symmetries are discussed in another paper [13]. Future work will examine the problem of ignoring features to find the regularity.

5 Survey Results and Discussion

The data for the survey were CAD models obtained from various WWW repositories [26, 30, 27, 21] viewed on a computer screen, or schematic diagrams on paper obtained from company catalogues [29, 22, 23, 28]. The broad classes of object surveyed were small engine parts (such as cog wheels and spindles), fittings and brackets for optical systems (see Figure 3), plastic fittings, caps and connectors (see Figure 9), sliding fittings for cupboards and the like, and a general selection of CAD display models from repositories (see Figure 10).

Exemplar	Frequency
Rectangular prism	12.4%
Square prism	10.8%
Cylindric	3.0%
Hexagonal prism	0.3%
Octagonal prism	0.3%
Pentagonal Prism	0.3%
2-fold saw prism	0.2%
3-fold saw prism	0.2%
4-fold saw prism	0.2%
Triangular Prism	0.2%
Two bladed fan	0.2%

Table 6: Prismatic Regularity Frequencies

Exemplar	Frequency
A rectangular tray	22.1%
Cone	12.9%
Square pyramid	4.8%
Hexagonal pyramid	0.9%
Triangular pyramid	0.5%
Octagonal pyramid	0.5%
2-fold saw pyramid	0.3%

Table 7: Pyramidal Regularity Frequencies

Regularity	Frequency
Cubic	1.0%
Single mirror only	8.3%
Pie wedge	0.5%

Table 8: Other Regularity Frequencies

Regularity	Bilateral	Frequency
Extrusion	yes	9.9%
2 in a row	no	1.8%
3 in a row	no	1.6%
2 in a row	yes	0.7%
4 in a row	no	0.5%
5 in a row	no	0.5%
6 in a row	no	0.5%
7 in a row	no	0.3%
8 in a row	no	0.3%
9 in a row	no	0.3%
10 in a row	no	0.3%
11 in a row	no	0.3%

Table 9: Linear Regularity Frequencies

The parts surveyed were chosen because they could be reverse engineered without having to know the design intent. Some characteristics of parts, such as the draught angle on a piece so that it can be removed from a mould, were ignored. The difficulty of machining a part was not considered; some parts with few faces are difficult to machine, others with many are fairly straight forward. We chose a limit of 200 faces as being a reasonable maximum complexity for a reverse engineering algorithm. Already we have preliminary algorithms for detecting regularities which can handle objects of such complexity in a few minutes. During the survey no parts where rejected for exceeding the maximum number of faces. We only rejected parts for having free form surfaces, and deep cavities which a laser scanner could not probe.

The classic Gehäuse benchmark part has 89 faces (see Figure 11). It is considered to be a reasonably complicated part and falls well within the range of the sort of thing that we considered. A typical example of the more complicated parts that we considered in the survey was a heat sink base for integrated circuit components (see Figure 12). It has 88 faces. Another was a pulley wheel with 53 faces. There were 600 parts examined, so we cannot list each part individually, but these two examples give a representative illustration of the type of part being examined. The extremum was a Boeing aircraft part, with 122 faces (see Figure 10). Clearly this survey is not exhaustive or complete, nor could *any* survey claim to be. Nevertheless we hope that it does give some insight into the nature of regularities to be found in reasonable range of objects of medium complexity which might be realistically reverse engineered.

Each part from each data set was briefly examined to determine whether it was suitable for scanning and modelling (the criteria are expressed further below). If so it was included in the survey. Only those parts that were clearly impractical were excluded (e.g. parts with deep concavities, see later). Each included part was examined and a decomposition of the body into negative and positive features was performed by hand. The body was examined for regularities and the features for constraints. Regularities and constraints that seemed accidental were excluded. The number of parts judged as having each type of regularity, constraint or feature was counted. The survey frequencies are intended as a lower estimate of the actual occurrence to justify the use of the regularities, constraints and features in beautification. Some ambiguity of definition is therefore tolerable, and only non controversial examples were included. The estimated frequencies are shown in Tables 2 to 9. If a regularity does not occur in the table, then it was not observed in the data set (e.g. 13-fold rotational symmetry).

Table 6 shows the prismatic symmetries; the final entry *two bladed fan* refers to a second order anti-prismatic symmetry on a base without mirror symmetry. Table 7 shows the pyramidal symmetries; a *saw* pyramid is one on a base without mirror symmetry, and the entry *tray* refers to a second order pyramidal symmetry. Table 8 shows the remaining symmetries. Most of the Platonic solids did not occur at all, and only the continuous version of pieces placed at regular angles occurred. Table 9 shows the various orders of linear repetition that occurred. In particular the limiting case of an extrusion was the most common. Table 4 shows a variety of axis configurations seen in the data. Table 2 shows both linear axes and the circular analogues (such as the central circle of a torus). A mixed constraint is between things that exist in the diagram, such a the edge of a face, and things that were derived, such as the diagonal of a face, or the axis of a cylinder. Table 3 shows simple shape constraints. Table 5 shows the occurrence of the six features we looked for. It is noted that the negative features are all more common than the positive features.

5.1 Suitable Parts

In the envisaged reverse engineering system the object is scanned in by illuminating a point on the surface with a laser and detecting its position with two cameras [11]. The laser scans quickly back and forth across the surface, and is slowly swept along a path, to input 3D points along a narrow strip of the surface. Certain types of parts are difficult to scan. The laser light must be able to reach each point of the surface and also be in view from both cameras. For practical purposes the whole surface should be obtained from a relatively small number of sweeps. Overhanging surface features and deep holes (greater than about 30mm) cannot be scanned.

The part also has to be physically small enough to be put on the scanner, which means that it must fit inside a 500mm cube, be light enough to man-handle onto the scanner bed, and its features must be large enough, (bigger than about 5mm) to provide enough data to be able to properly fit surfaces.

5.2 The Parts Survey

The total number of parts surveyed was about 600 and 97% had strong regularity. We attempted to survey a fairly wide range of part types. Nevertheless, clearly our survey has some limitations. Only some of the companies polled returned catalogues, and the WWW repositories were mostly biased towards demonstrations of modelling prowess. Further, the frequency of occurrence of regularities, features and constraints in catalogues is likely to be different from that of parts submitted for reverse engineering. However, the bias would be towards a greater variety, and complexity of parts, which would tend to reduce the measured regularities. As long as irregular objects are less than about 30 times more likely to be picked for reverse engineering, the conclusion stands that regularities occur more often than not.

Some of the regularities, features and constraints are common, some are not common, and some were not observed to occur in the test set at all. Rotational symmetry above eight-fold, for example, was not observed, and five- or seven-fold were both very rare. In both constraints and regularities there is a steep drop off of observed frequencies in the middle of the tables, separating the particularly common from the fairly unusual. This information will be useful in algorithm design. If some rare aspect turns out to be difficult to determine or enforce, then it can safely be left out of an initial implementation of beautification.

The majority of interesting aspects of objects turn out to be strongly related to cubes, or at least to rectangular prisms. The major symmetries involved are prismatic and pyramidal of orders 2, 4 and 6, all of which can be produced by decorating cubes. The only common exceptions are conic and cylindrical symmetry. Identity of things such as centres, and equality of lengths and radii turn up a lot more than other ratios such as 1:2 or 1:3. Angles of $\pi/2$ and $\pi/4$ turn up more often than others, and orthogonal systems that include central axes are also fairly typical. All of these aspects are related to decorating cubes.

Regularities, constraints and features are logically related in that some combinations of these imply or exclude the existence of others. Beyond this there is little evidence of statistical correlation. Features, being local, could in principle occur in any combination, and constraints dealing with multiple faces do not very strongly interfere with constraints dealing with other faces.

The regularities as listed are mostly but not entirely exclusive of each other so the percentages add to approximately 100. However, the features and constraints are much more independent and add up to about 1000 in our observations, illustrating the heavy overlap.

5.3 Conclusion

This report justifies the expectation that the listed regularities exist with sufficient frequency in order to be used to write better beautification algorithms in practice. Doubtful parts were included, and doubtful regularities excluded, which together bias the results towards the negative. Overall, as expected, there is strong evidence that certain constraints, features and regularities are of sufficiently frequent occurrence that it is worth attempting to beautify these in reverse engineering systems.

This survey has established a qualitative classification of regularities and symmetries into rare, common, and in-between. We learn from this what it is that we will need to beautify. This allows us to concentrate on algorithms dealing with regularities that will be found in practice, and also allows the overall process to make decisions. If we find some regularity, but it is an unlikely one, then we may chose to drop it out, or choose to give it a lower priority in subsequent enforcement algorithms. This report provides source data for the weightings for these priorities. The exact percentages are not especially meaningful, but the approximate percentages are very useful to know.

Current work on this project is considering the detection and imposition of characteristics of the model. We are investigating appropriate methods of detecting constraints, regularities and features when only approximately present [7, 8, 13]. The degree to which these are satisfied can be improved by least squares solution of the related system of equations in terms of the parameters of the model. We will investigate this approach, but, since it tends to be approximate, and to destroy already satisfied constraints, we will also investigate other methods of optimising the characteristics. It is expected that the grouping of constraints, and the satisfaction of those groups in sequence will be a significantly more adaptable strategy than the imposition of all the constraints simultaneously. However, methods for meaningfully grouping and prioritising must be developed.

Existing geometric constraint systems [24] typically consider the case where the number of constraints is, at least in principle, just sufficient to define the object, and are defined by the user. In contrast, we expect to have far more constraints that the minimum number needed, and it is highly likely that they will not all be consistent. Thus, new methods will be required to select and enforce the most useful subset.

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