Finding and Characterising Robust Quantum Controls

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Background

Quantum control (QC) provides methods to steer the dynamics of a quantum device, e.g., in order to implement quantum gates, prepare quantum states or transfer information. Most quantum control algorithms follow an open-loop control design, optimising a fidelity target function using a device model in simulation [1]. Some approaches replace the simulation with the real device [2,3] but most focus on achieving very high fidelity operations. In quantum computing and communication, given sufficiently high accuracy, experimentally and in simulation, quantum error correction schemes can then ensure reliable operation, if one has sufficient physical qubits. More generally, robust operations are important for quantum technologies as a whole.

Current promising scalable quantum computing architectures (e.g. transmons, trapped atoms or ions) are noisy and subject to a range of uncertainties that are expensive to include in the control simulation models, and even harder to optimise for to achieve high accuracies experimentally. These effects can be somewhat mitigated via the use of algorithms that can cope with noisy models [3], reinforcement learning approaches [4,5], and optimization in the actual experimental system [6,7]. These also include manually engineering Hamiltonians to account for small linear perturbations and achieve “noise cancellation”, or numerically optimizing for metrics assuming some form of the noise. Examples of the former approach include treating the qubit phase as a topological winding number to engineer a two-level analytic driver with noise-cancellation capability [8] or expanding a perturbation to first order to engineer composite single qubit pulses [9].

QC, unlike its classical counterpart, generally lacks explicit concepts of robustness of the controller, i.e., the ability of the system to achieve high fidelity operations under uncertain dynamics and state preparation errors. If considered at all, robustness is typically explored via Monte Carlo (MC) simulations for certain device uncertainties or experiments [10, 11]. Among the analytical approaches, the quantum control landscape has been extensively studied using techniques such as spectral analysis of the Hessian of the fidelity functional with respect to control parameters [12]. In this context, robustness is related to the flatness of the landscape, characterized by the negative and zero eigenvalues of the Hessian. However, none of these approaches are entirely satisfactory, nor do they fully capture robustness the way classical control does. Unfortunately, the setting for most of classical robust control of linear, time-invariant systems stabilized by feedback, and robustness measures based on signal processing filter functions and frequency-domain sensitivity analyses, is not well suited to many QC problems [13].

Novel tools and approaches for robust quantum control and related fundamental limitations are therefore needed. In addition to uncertainties in the control parameters, a wider range of uncertainties in the system have to be considered. New measures to quantify robustness and new quantum control approaches capable of explicitly finding robust controls are needed.

Presentation

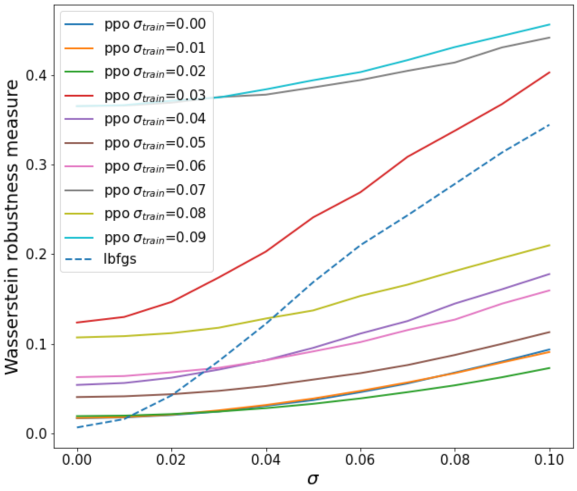

Fig. 1: Wasserstein robustness measure for a 5 spin chain for controlled transfer from spin 1 to 3. We compare controllers found with reinforcement learning at different training noises σ_{train} using proximal policy optimization (PPO) at various Hamiltonian uncertainty strengths and also controllers found using L-BFGS without any added noise. The average robustness for 100 controllers for increasing MC simulation noise σ from 0 to 0.1 in steps of 0.01 is shown.

We present results of our recent efforts in addressing these questions. A new robustness measure that combines fidelity and its variance under uncertain system dynamics and initial state preparation error, based on the Wasserstein distance [14], is introduced. It is used to assess the robustness of controllers computed with different algorithms, including gradient-based control algorithms, designed for optimising ideal, perfectly known system dynamics, and reinforcement learning approaches that aim to optimise system dynamics without any prior knowledge of the system. Exploiting the degree of freedom afforded by the existence of many optima in the QC problems, we explore the control space to identify a large number of candidate solutions, which are then assessed with regard to robustness properties. The approach can be applied to a wide range of QC problems, including dynamic open loop control, but we specifically apply it to study information transfer in spin-1/2 rings and chains subject to energy-landscape control. While such systems are in general not (fully) controllable, previous work has shown that solutions can nevertheless be found easily [15]. Moreover, such controls can be mathematically modelled as feedback control laws that do not only show similar robustness properties relative to conventional direct feedback systems but exhibit these at optimal performance (globally maximal fidelity) of the operation [16].

To measure robustness, we consider various sources of errors, especially uncertainties modelled by structured perturbations δS to the Hamiltonian where δ is the strength and S the structure that indicates the parameters of the Hamiltonian affected by the perturbation [17]. The uncertainty is described by two probability distributions for δ and S. Specifically, we use a simple noise model structure S, where the local potential and nearest neighbour couplings in the Hamiltonian are all equally uncertain. δ is given by a Gaussian $N(0, \sigma^2)$. MC simulations (or experiments) generate a probability distribution $P_{\delta,S}(f)$ for the fidelity f. Based on ideas from probabilistic optimal transport, we propose a robustness measure, the (first-order) Wasserstein distance between $P_{\delta,S}(f)$ and the perfectly robust distribution, a Dirac delta function at $f = 1$. It measures, both, fidelity and variance under uncertainty, and imposes a metric on the controller space. Robustness of a controller is computed purely statistically using synthetic MC sampling of the Hamiltonian space according to the distributions for δ and S.

We consider the control problem in the single excitation subspace of spin chains and rings to evaluate the viability of the robustness metric. Results indicate the utility of our robustness measure and consistency with other easily computable measures like worst-case fidelity, standard deviation, above x-fidelity-threshold percentage. We further study the robustness of controllers found under various noise levels with L-BFGS and reinforcement learning. Fig. 1 shows that while L-BFGS finds high accuracy controllers under perfect, no-noise conditions, our work indicates that these are not very robust, as their fidelity and robustness quickly drop as noise is added. Controllers found with RL are comparably more robust as noise increases and it seems that there is an optimal noise level at which particularly robust controllers can be found; in the plot this is indicated by the green curve ($\sigma_{train} = 0.02$) where the robustness measure is most slowly increasing as a function of the noise.

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