Finding and Characterising Robust Quantum Controls

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Summary

1. To quantify robustness and infidelity of quantum controllers together, a **statistical Monte Carlo robustness-infidelity measure** (**RIM**) is presented with **tunable structured perturbations**.
2. Analogously, a measure to describe the performance of algorithms to find high-fidelity, robust controllers is presented as **algorithmic RIM** (**ARIM**).
3. Numerical results on energy landscape control of **XX-Heisenberg chains** indicate that not all high-fidelity controllers are also robust (see Fig. 3).
4. There exists some benefit of incorporating **certain noise in finding low RIM controllers** (see Fig. 4).

Quantum Control Problem

The state transfer optimization problem is given by

$$t^*, \mathbf{\Delta}^* = \underset{t,\mathbf{\Delta}\in\mathbb{T}}{\arg\max} \underbrace{|\langle \psi^* | U_c(t,\mathbf{\Delta}) | \psi^i \rangle|^2}_{=:\mathcal{F}} \quad (1)$$

where $U_c = \exp(-iH_c(\mathbf{\Delta})t)$ and the **XX Heisenberg Hamiltonian** $H_c(\mathbf{\Delta})$ of the L-body chain with energy landscape controls $\mathbf{\Delta}$ in the single excitation subspace is [1]:

$$\frac{H_c(\mathbf{\Delta})}{\hbar} = \sum_n^L \left(\Delta_n |n\rangle\langle n| + J|n\rangle\langle n \pm 1|\right) \quad (2)$$

We **structurally perturb** the coherent dynamics by adding Hamiltonian noise: $H_c \to (\mathbb{I} + S_{\sigma_{\text{sim}}}) \odot H_c$ using

$$(S_{\sigma_{\text{sim}}})_{l,m} = J(p_1\delta_{l,m\pm1}) + p_2\Delta_{l,m}\delta_{l,m}, \quad p_1, p_2 \sim \mathcal{N}(0, \sigma_{\text{sim}}) \quad (3)$$

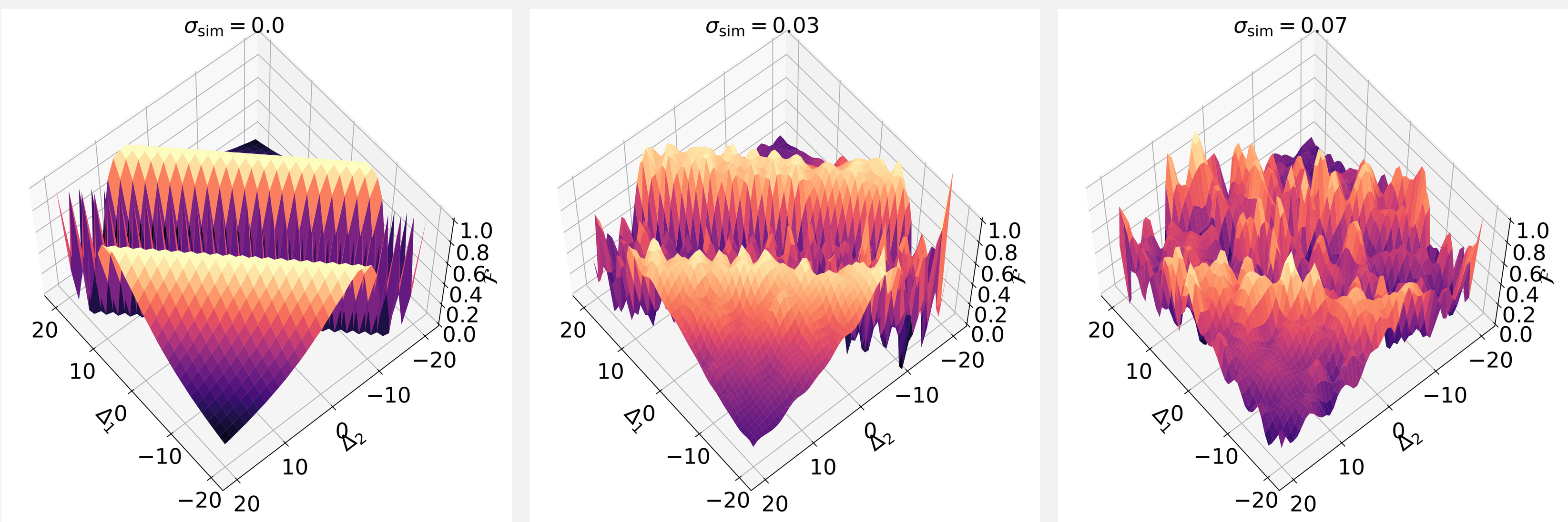

Figure 1: Fidelty $\mathcal{F}$ landscape (Eq. (1); $\Delta_1 = \Delta_3$) with one structured perturbation (Eq. (3)) for $L = 3$ and an end-to-end transition.

Robustness-Infidelity Metric

The robustness-infidelity metric (RIM) is the first order Wasserstein distance of a **fidelity distribution** $\mathcal{P}_{\sigma_{\text{sim}}}(\mathcal{F})$ from the perfectly robust fidelity distribution $\delta(\mathcal{F}-1)$ induced by the uncertainty $S_{\sigma_{\text{sim}}}$

$$\text{RIM} := \mathcal{W}(\mathcal{P}_{\sigma_{\text{sim}}}(\mathcal{F}), \delta(\mathcal{F}-1)) = \int_0^1 dx\, \Pr(\mathcal{F} \le x) \quad (4)$$

1. Works on any bounded fidelity measure $\mathcal{F}$.
2. Captures statistical robustness w.r.t. $S_{\sigma_{\text{sim}}}$ and the fidelity $\mathcal{F}$ as a single measure.
3. RIM is 0 iff a controller parametrizing an optimum for Eq. (1) is perfectly robust and has perfect fidelity. So it generalizes fidelity to also capture robustness. Moreover, $\sigma_{\text{sim}} = 0$ implies that RIM $= 1 - \mathcal{F}$ is the infidelity.
4. Can be used for controller post selection.
5. The first order Wasserstein distance is a metric on the space of controllers that facilitates robust optimization

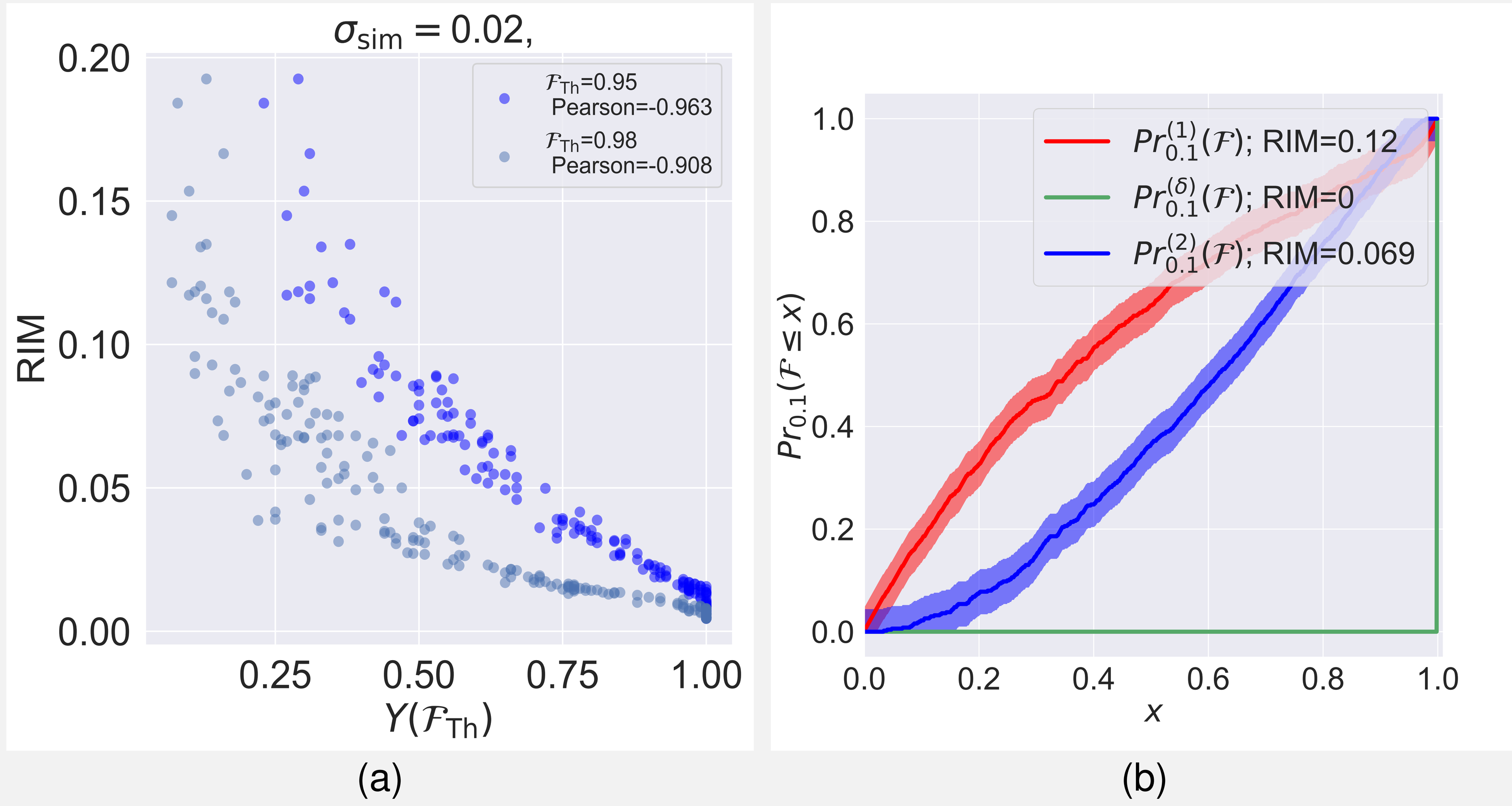

Figure 2: (a) Comparison of RIM with yield Y (fraction of fidelities greater than threshold fidelity $\mathcal{F}_{\text{th}}$) at values $\mathcal{F}_{\text{th}} = 0.95, 0.98$ for 200 controllers. (b) Illustration of how RIM is calculated for a single controller. Both figures are generated for a chain of length $L = 5$ and a bit transition from $|1\rangle$ to $|3\rangle$.

References

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Results

Individual controller comparison using RIM for chain of length $L = 5$ and transition from $|1\rangle$ to $|3\rangle$. Controllers are obtained numerically using L-BFGS ($\sigma_{\text{train}} = 0$) [3, 1], PPO [4] and SNOB-Fit ($\sigma_{\text{train}} = 0, 0.01, \ldots, 0.05$) [5]. Performance of controllers is evaluated at uncertainty strengths $\sigma_{\text{sim}} = 0, 0.01, \ldots, 0.1$.

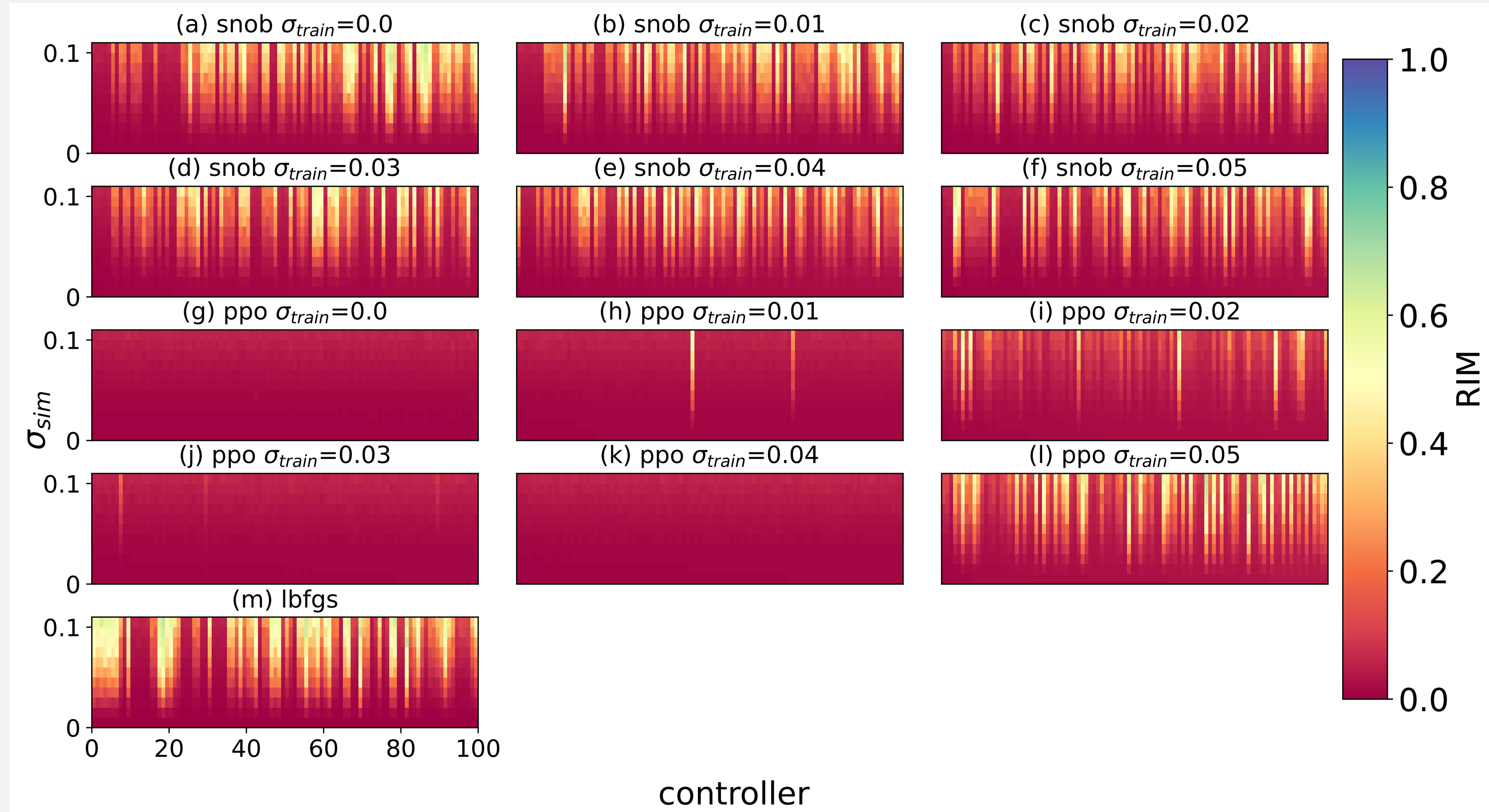

Figure 3: Top 100 controllers sorted (left to right) by $1 - \mathcal{F}$ for the uncertainty level $\sigma_{\text{sim}} = 0$.

Algorithmic RIM (ARIM) for RIM distribution comparison,

$$\text{ARIM} := \mathcal{W}(\mathcal{P}_{\sigma_{\text{sim}}}(\text{RIM}), \delta(\text{RIM} - 0)) \quad (5)$$

ARIM inherits properties of RIM. It can be used to compare empirical controller acquisition schemes and indicate the performance of different algorithms w.r.t. finding robust, high-fidelity controllers for a specific control problem.

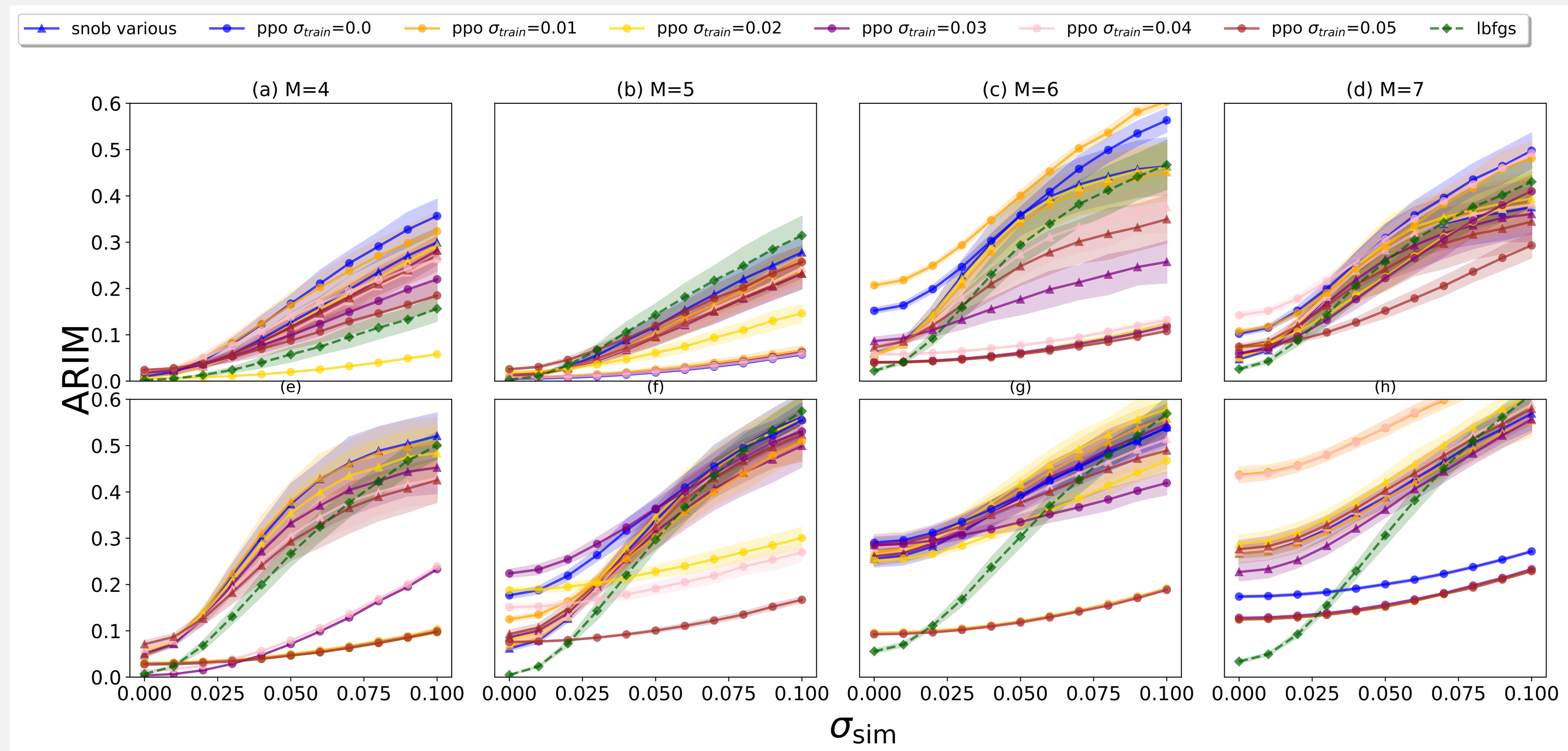

Figure 4: ARIM comparison for transitions from one end to the middle (a)-(d) and to the end (e)-(h) for $L = 4, 5, 6, 7$ (columns) for the top 100 controller RIMs found by different algorithms.